



Symmetry energy determination in heavy ion collisions

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In this talk, we will examine isoscaling, the exponential dependence of the isotope yield ratios on the neutron number and proton number, as an experimental observable to explore the density dependence of the symmetry term in the nuclear Equation of State.

1. SYMMETRY ENERGY FROM NUCLEAR MASSES

Treating nuclei as liquid drops has been very successful in predicting their properties and masses, especially those along the valley of stability. The most commonly used Liquid Drop Mass (LDM) parametrization [1] involves the following formula for the nucleus binding energy:

$$BE_{LDM}(A,Z) = a_v A - a_s A^{2/3} - a_c Z^2/A^{1/3} + a_p A^{-1/2} - a_{sym} (N-Z)^2/A \quad (1)$$

where N , Z and A are the neutron, proton and mass numbers; a_v , a_s and a_c are the volume, surface and Coulomb terms in the liquid drop model. $a_p A^{-1/2}$ is the pairing energy. The last term in Eq. (1), $a_{sym} (N-Z)^2/A$, accounts for the symmetry energy which favors nuclei with equal number of protons and neutrons. In most parametrizations, a_{sym} is a constant set to about 24 MeV. In this formulation, the symmetry energy is independent of the nucleon density.

It has been pointed out by the study of Myers-Swiatchi [2] that the agreement of the calculated masses with experimental masses can be improved if the LDM formula includes corrections to the surface term due to asymmetry. The parametrization of the improved LDM formula is given in ref. [3]:

$$BE_{ILDLM}(A,Z) = a_v [1 - k((A-2Z)/A)^2] A - a_s [1 - k((A-2Z)/A)^2] A^{2/3} - a_c Z^2/A^{1/3} + a_p A^{-1/2} + C_d Z^2/A \quad (2)$$

where the extra Coulomb term $C_d Z^2/A$, neglected in most models, takes into account corrections to the Coulomb energy associated with the diffuseness of the nuclear surface. The symmetry terms in Eq. (2) can be regrouped in a form similar to Eq.(1)

$$a_{sym}' = k (a_v - a_s A^{-1/3}) \quad (3)$$

Contrary to the simple liquid drop model, the nucleon number dependence of the symmetry coefficient reflects the density dependence of the asymmetry term of the

nuclear equation of state [2,4]. For very neutron-rich nuclear matter the surfaces can accumulate a significant fraction of asymmetry [2,4]. The best fit parameters of Eq. (2) to empirical nuclear binding energy [5] gives $ka_v = 27.7976$ MeV and $ka_s = 33.7053$ MeV.

2. SYMMETRY ENERGY FROM HEAVY ION COLLISION STUDY

The nuclear equation of state (EOS) is often approximated by a parabolic function as a function of the nuclear density and asymmetry, $\delta = (\rho_n - \rho_p) / \rho$ [6]

$$E(\rho, \delta) = E(\rho, 0) + E_{\text{sym}}(\rho) \delta^2 \quad (4)$$

Recently, progress has been made in placing significant constraints on the symmetric matter EOS, $E(\rho, 0)$ [7]. Since the density dependence of the asymmetry term of the nuclear Equation of State (EOS) affects the neutron star radii, matter distributions, moments of inertia, and the cooling rates of proto-neutron stars [6], these new results show that current uncertainties in the isospin symmetry terms of the EOS may contribute much less to the overall uncertainty in neutron star characteristics than do the corresponding uncertainties in the density dependence of the asymmetric term [7].

Availability of rare isotope beams provide opportunities to prepare and study the dynamical evolution of nuclear systems with a range of isospin asymmetries, characterized by δ approximated by $(N-Z)/(N+Z)$. Such systems can be prepared using beam and target pairs of similar isospin asymmetry. This increases the probability that a spatially uniform local isospin asymmetry may be achieved and renders the systems more suitable for investigating the density dependence of the asymmetry term of the nuclear equation of state. Alternatively, systems can be prepared using beam and target pairs with very different isospin asymmetries. This isospin "tagging" permits more sensitive tracking of the transport and diffusion of neutrons and protons during collisions, information that is key to testing and refining theoretical models of such collisions.

To extract information about the density dependence of the asymmetry term, we use fragment isotopic yield data from four collisions at $E/A = 50$ MeV: $^{112}\text{Sn} + ^{112}\text{Sn}$, $^{112}\text{Sn} + ^{124}\text{Sn}$, $^{124}\text{Sn} + ^{112}\text{Sn}$, $^{124}\text{Sn} + ^{124}\text{Sn}$ [8,9]. This method involves comparisons of isotope yields from two reacting nuclear systems of similar mass, similar excitation energy, but different charge-to-mass ratio [9]. The collisions of Sn isotopes are studied by bombarding ^{112}Sn and ^{124}Sn targets of 5 mg/cm^2 areal density with 50 MeV per nucleon ^{112}Sn and ^{124}Sn beams from the K1200 cyclotron in the National Superconducting Cyclotron Laboratory at Michigan State University. Isotopically resolved particles with $1 \leq Z \leq 8$ were detected with nine telescopes of the Large Area Silicon Strip Array (LASSA) located at a polar angle of $\theta = 32^\circ$ with respect to the beam axis, covering polar angles of $7^\circ \leq \theta \leq 58^\circ$. Impact parameter selection was provided by the multiplicity of charged particles measured with LASSA and with 188 plastic scintillator - phoswich detectors of the Miniball/Miniwall array. Details of the experiment are described in ref. [8].

In general, the ratios of the measured isotopic yields $Y_1(N, Z)$ and $Y_2(N, Z)$ of two reactions (1 and 2), which occur at the same center of mass energy per nucleon, obey an "isoscaling" law

$$R_{21}(N,Z) = Y_2(N,Z) / Y_1(N,Z) = C \cdot \exp(\alpha N + \beta Z) \quad (5)$$

where C is a normalization constant, α and β are isoscaling parameters in the two reactions and N , Z are the neutron number N and proton number Z of the isotope. In the left panel of Figure 1, the open (^{112}Sn projectile) and closed (^{124}Sn projectiles) data points show the linear dependence of the neutron isoscaling parameter, α , on the asymmetry of the initial system, δ , for the central collisions ($b > 0.8b_{\text{max}}$) of the four reactions; $^{112}\text{Sn} + ^{112}\text{Sn}$ ($\delta = 0.107$), $^{112}\text{Sn} + ^{124}\text{Sn}$ ($\delta = 0.153$), $^{124}\text{Sn} + ^{112}\text{Sn}$ ($\delta = 0.153$) and $^{124}\text{Sn} + ^{124}\text{Sn}$ ($\delta = 0.193$). We use the convention that reaction 2 refers to the more neutron-rich system and the reference reaction 1 is taken to be $^{112}\text{Sn} + ^{112}\text{Sn}$. To minimize contributions from projectile fragments, a center of mass angular gate of 70° to 110° has been imposed. Previous study of similar system suggests that in central collisions, the system undergoes bulk multifragmentation and the isoscaling behavior is predicted by grand canonical multi-fragmentation theories [9]. In these approaches the isoscaling parameters, α and β are related to the temperature and differences in the neutron and proton chemical potentials for the two systems. Microcanonical and canonical calculations also respect the isoscaling law as discussed in Ref. [10]. Generally, one expects the isoscaling parameters to depend on the isospin asymmetry and excitation energy per nucleon of the system undergoing multifragmentation, both of which evolve considerably prior to the multifragment breakup [9,10].

A hybrid approach calculates fragmentation observables with a statistical model using a source determined by a dynamical model. This approach is frequently employed to calculate fragmentation observables. Two different statistical approaches have been attempted for this latter stage. In the Expanding Emitting Source (EES) formalism, the fragments are emitted sequentially from the surface of the system as it expands [11]. Sensitivity to the density dependence of the asymmetry term arises because the separation energies of the fragments from the remainder of the expanded system depend on the symmetry energy [11]. Predictions for α assuming an asymmetry term of the form $E_{\text{sym}}(\rho)\delta(r)^2$, where $E_{\text{sym}}(\rho) = 23.5(\rho/\rho_0)^\gamma$ MeV [9] gives the best agreement with the data for $\gamma = 2/3$, corresponding to a relatively soft asymmetry term in the EOS.

If one assumes, alternatively, that equilibrium is achieved during the breakup and models the multifragment breakup by an equilibrium approach such as the Statistical Multifragmentation Model (SMM) [12], a different picture emerges. Calculations of the multi-fragment decay of this prefragment were performed in ref. [13], and a preference for an asymmetry term with a much stronger density dependence, approximately $(\rho/\rho_0)^2$ dependence was indicated by comparing these calculations to the data [13].

These different predictions reflect different assumptions about the fragmentation mechanism. It is thus imperative that another approach be used to study the symmetry terms in the equation of state of nuclear matter. At low incident energies, the protons and neutrons can be redistributed on a time scale that is faster than the typical timescales for decay. In this sense, one can say that particle emission occurs from a system that is in “isospin equilibrium”. As the incident energy is increased above $E/A = 30$ MeV, however, the time scale for emission decreases and anisotropies in the emission patterns may

develop, which allows one to measure the time scales for charge and mass transport and diffusion during the collision. Peripheral collisions provide an excellent environment for probing these time scales. Boltzmann-Uehling-Uhlenbeck (BUU) calculations suggest that at large asymmetries, the asymmetry term of the nuclear EOS provides a significant driving force that speeds up the isospin equilibrium.

Peripheral collisions from the four Sn+Sn reactions discussed earlier were selected by gating on the charged-particle multiplicity distribution, corresponding to a reduced impact parameter of $b/b_{\max} \geq 0.8$ in the sharp cut-off approximation. Events from two different regions of the collisions are studied. 1). Neck fragments data were selected by a center of mass angular gates of $70^\circ \leq \theta_{\text{CM}} \leq 110^\circ$. 2). Projectile like fragments were selected by a rapidity gate of $y/y_{\text{beam}} \geq 0.7$, where y and y_{beam} are the rapidities of the emitted particle and beam, respectively.

We first use the symmetric $^{112}\text{Sn} + ^{112}\text{Sn}$ and $^{124}\text{Sn} + ^{124}\text{Sn}$ collisions to infer what should be the final isotopic distributions from projectile decay in the absence of diffusion between projectile and target since there should be no isospin gradients between the projectile and target of the same isotopes. The isospin gradients between the projectile and target in the asymmetric $^{124}\text{Sn} + ^{112}\text{Sn}$ and $^{112}\text{Sn} + ^{124}\text{Sn}$ collisions are used to assess the isospin diffusion. The open and closed squares in the left panel of Figure 1 shows the dependence, α , as a linear function of $\delta \approx (N-Z)/(N+Z)$ of the initial collision systems for the neck fragments. Within experimental uncertainties, the extracted values of α for the two asymmetric systems ($\delta=0.153$) are the same, reflecting the quality of the data analysis since the center of mass gates around 90° should be equivalent for $^{112}\text{Sn} + ^{124}\text{Sn}$ and $^{124}\text{Sn} + ^{112}\text{Sn}$. The differences in the slope of the α values for the peripheral and central collisions reflect the differences in temperature and density of the disintegrating systems. Most likely, complete isospin mixing occurs in the central collisions. The current results thus suggest that isospin of the neck fragments may have achieved equilibrium.

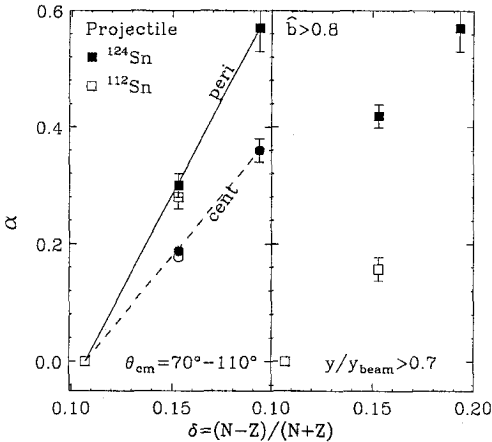


Figure 1: Isoscaling parameter, α , plotted as a function of the asymmetry δ for central (circles) and peripheral (squares) collisions.

The α values extracted from the projectile like fragments with a gate on the rapidity, $y > 0.7y_{\text{beam}}$, are plotted in the right panel of Figure 1. The two symmetric systems ($^{112}\text{Sn}+^{112}\text{Sn}$; $\delta=0.107$ and $^{124}\text{Sn}+^{124}\text{Sn}$; $\delta=0.193$) have similar α values as those extracted from the neck fragments. However, the extracted α values of the two asymmetric systems ($\delta=0.153$) are different thus breaking the linear dependence of α on δ shown in the left panel. The open symbols refer to the system with the light ^{112}Sn projectile and the closed points denote collisions using ^{124}Sn projectile. The α value from the $^{124}\text{Sn}+^{112}\text{Sn}$ reaction is larger than that obtained from $^{112}\text{Sn}+^{124}\text{Sn}$ reaction, reflecting the memory of the isospin composition of the projectile. In the limit of complete isospin equilibration (as shown in the left panel), the extracted α values for the projectile decay in the $^{112}\text{Sn} + ^{124}\text{Sn}$ system is about what one would expect for symmetric $^{115}\text{Sn} + ^{115}\text{Sn}$ ($\delta=0.130$) collisions. Similarly, the result from the projectile decay of $^{124}\text{Sn} + ^{112}\text{Sn}$ collisions is what one would expect for symmetric $^{121}\text{Sn} + ^{121}\text{Sn}$ ($\delta=0.174$) collisions. This simple observation suggests movement of 3 neutrons from the ^{124}Sn nucleus to the ^{112}Sn nucleus in the absence of pre-equilibrium emissions.

Differences in the asymmetric systems reflect the driving force arise from the asymmetry terms in the equation of state. If the force is weak, one would not expect any isospin mixing to occur and the α values should resemble those of the projectiles in the symmetric systems. In order to quantify this transition from no isospin diffusion to complete mixing, we define an isospin transport ratio [14],

$$R_i = \frac{2\alpha - \alpha_{124+124} - \alpha_{112+112}}{\alpha_{124+124} - \alpha_{112+112}} \quad (6)$$

where α is the neutron isoscaling fitting parameter of Eq. (5).

Equation (6) allows quantitative comparison with model calculations using isospin observable that can be predicted but not measured experimentally. For example, using δ , the asymmetry of the projectile-like fragments predicted in the BUU transport model [15, 16], we can compare to the experimentally obtained isoscaling parameters. Preliminary results suggest that the data is in better agreement with the predictions if a stiff symmetry term in the EOS, approximately $E_{\text{sym}}(\rho)=12.125(\rho/\rho_0)^2$ MeV, is used than a softer symmetry term [16].

3. SUMMARY

Figure 2 provides an pictorial summary of the current status of using isoscaling as observable to study isospin mixing and isospin diffusion both of which are affected by the symmetry terms in the nuclear EOS. In this figure the isospin transport ratios, R_i (y-axis) of the data in Figure 1 from peripheral collisions are plotted against the absolute values, $|R_i|$ (x-axis). By definition of Eq. (6), R_i of the symmetric collisions of $^{112}\text{Sn}+^{112}\text{Sn}$ and $^{124}\text{Sn}+^{124}\text{Sn}$ equal to -1 and 1 (open and closed circles), where isospin diffusion should not occur. The data for the asymmetric Sn isotope collisions, where isospin diffusion is expected to occur, are plotted as open and

closed squares, suggesting nearly complete isospin equilibrium for the neck fragments and the incomplete mixing of the projectile like fragments. Further work, especially in theoretical understanding of the isospin transport observable is expected to provide constraints to the density dependence of the symmetry energy in nuclear equation of state.

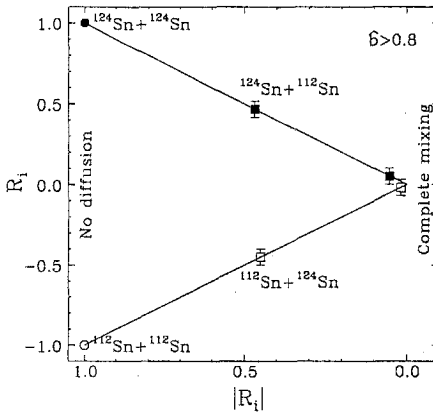


Figure 2: Summary of the peripheral collisions data plotted in terms of the isospin transport ratios determined from Eq. (6). See text for details.

This work is supported by the National Science Foundation under Grant Nos. PHY-0110253, INT-9908727 and INT-0124186.

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